QUESTION: I have some five-gallon cans that contain oil with a small amount of ^{56}Co (846 keV gamma) suspended in it. Because the total activity is low (<1 µCi), measuring the activity contained in the can is easier if I place my pancake probe close to the whole can rather than measuring a small sample from the can. Unfortunately, a can doesn't look much like a point source, and this makes the geometric factors needed to calculate activity complex. Is there an accepted formula for cylindrical containers (such as barrels) to back-calculate activity? Is there a standard protocol for where the measurements are done-on the side at the midpoint of the oil level? Above the top of the can? Both?

ANSWER: Unfortunately, there are, to my knowledge, no simple expressions that apply to the determination of photon fluence rates or dose rates close to the surfaces of a cylindrical volume source, except for the situation when the distance from the dose point to any source boundary is significantly greater than the mean free path of photons contributing to the dose; for this special case we can invoke a principle referred to as energy spatial equilibrium (ESE) in which we take advantage of the fact that the decay energy emitted per unit mass of source material is equal to the energy absorbed per unit mass. For such a case when the dose point is at the surface of the source the ESE dose rate is divided by two to account for the fact that source material is present on only one side of the dose point. The number of mean free paths is given by the product of the linear attenuation coefficient and the linear distance from source point to dose point.

For ^{56}Co in oil in a typical five-gallon bucket the source dimensions are not large enough to produce a state of ESE in the source volume, the usual five-gallon container having an approximate radius of 5.5” (14 cm) and a (liquid) depth of about 12” (30.5 cm). ^{56}Co emits gamma rays covering a wide range of energies with the 846 keV photon being one of the more significant gammas. The linear attenuation coefficient for these photons in a medium such as oil, with an assumed density of 0.92 g/cm^3, is about 0.072 cm^{-1}. Thus, the shortest distance from the center of the source to the container wall is 14 cm, and the number of mean free paths is then (0.072 cm^{-1})(14 cm) = 1.0. To apply ESE such distance preferably should be greater than 3 mean free paths.

For more general cases we can obtain a mathematical representation of the dose rate, but such representation will be in the form of an integral equation that requires solution through numerical integration or other approximation techniques. The simplest form of equation is obtained when the dose point lies along the central length axis of the cylinder and, in particular when the dose point is on the axis at the source surface. For such a case we can derive the integral equation that yields the dose rate at the dose point; it will be of the following form:

$$D = KS_v E \frac{\mu_{en}}{\rho} \int_0^R \int_0^H \frac{rBe^{-\mu\sqrt{r^2+h^2}}}{2(h^2+r^2)} dhdr \quad , \quad (1)$$

where D is the dose rate, rad/hr;

S_v is the gamma emission rate per unit volume, cm^{-3} s^{-1};
E is the photon energy of interest, MeV;
μ_{en}/ρ is the mass energy absorption coefficient for the photons of energy E in the source material;
r is the variable radial distance from the axis to the differential source volume element, cm;
B is the dose buildup factor for the photons of interest; the Taylor form was used here and is given by $B = A \exp(-\alpha_1\mu(r^2+h^2)^{0.5}) + (1 - A)\exp(-\alpha_2\mu(r^2+h^2)^{0.5})$, where A, α_1, and α_2 are constants for a given photon energy and material and were taken from a compilation by Foderaro (*The Photon Shielding Manual*, Penn State Book Store, now out of print), but values are available in a number of sources in radiation shielding;
μ is the linear attenuation coefficient for the gamma rays of interest, cm^{-1};
h is the variable depth along the axis from the dose point to the depth of the differential source volume element, cm;
R is the radius of the cylindrical source, cm;
H is the height of the cylindrical source, cm, and
K is a collective constant to get into desired dose rate units.

For the dose rate in rad/hr and other dimensions as given above, K has the numerical value of 5.76×10^{-5}.

As noted above, ^{56}Co emits multiple gamma rays. I have taken information from Kocher (*Radioactive Decay Data Tables – A Handbook of Decay Data for Applications to Radiation Dosimetry and Radiological Assessments*, Technical Information Center, US Department of Energy, Washington, DC; 1981). For simplicity I disregarded photons with yields of less than 1% and grouped other photons into two energy groups, the first from 0.846 MeV to 2 MeV, and the second from 2 MeV to 3.3 MeV. Parameter values used were determined for average energies for the two groups that were determined using the appropriate yields given in Kocher for each energy. Attenuation coefficients and mass energy absorption coefficients for water were used to simulate those for oil; the assumed water density was 0.92 g/cm^3. For calculational purposes, the assumed activity in the source volume was 1 μCi. Other parameter values used are summarized below:

Photon average energy	Photon yield	μ	μ_{en}/ρ	A	α_1	α_2	S_v
Group 1: 1.07	2.045	0.0644	0.031	19.6	-0.094	-0.025	4.00
Group 2: 2.66	0.416	0.043	0.0245	11.6	-0.042	+0.028	0.813

Equation 1 was applied separately to each of the above energy groups, and the integrations were carried out by numerical techniques using personal software. The dose rates in air were obtained from the dose rates in the source material by multiplying by the ratio of the respective mass energy absorption coefficient in air to that in water. Results are summarized below:

Group	Dose rate in source material, rad/hr	Dose rate in air, rad/hr
Group 1	6.8×10^{-5}	6.1×10^{-5}
Group 2	2.4×10^{-5}	2.2×10^{-5}
Total	9.2×10^{-5}	8.3×10^{-5}

Please note that the above calculations do not account for 0.511 MeV photons produced by annihilation of decay positrons in the source volume. The positron yield is 0.199 and would increase the Group 1 photon yield by almost 20%. This would result in about a 10 to 15% increase in the total dose rates in the table above.

For comparison, I also ran this case using the commercial computer code, Microshield (originally produced by Grove Engineering and now available from Framatome); the total air dose rate was 0.080 mrad/hr, not significantly different from the value above. The Microshield code handles all the significant energies (above 15 keV), but this made little difference in the results compared to the technique described. Since you will not be able to hold the detector directly on the source surface, I also used Microshield to obtain dose rates from the same source at some other heights on the source axis but above the source surface and obtained the following:

Height above surface, cm	Dose rate, mrads/hr
1.0	0.065
2.0	0.056
5.0	0.039
8.0	0.028

Note that for the assumed 1 µCi activity the dose rates are quite low, the highest being a few times likely background rates, and may be difficult to measure with a G-M detector. When making the measurements you should cover the face of the probe with a plastic or other low-atomic number absorber with sufficient thickness (250 mg/cm^2 should be sufficient) to stop any particulate decay radiation, notably positrons with energies approaching 650 keV. You should hold the probe face as close to the center of the flat surface of the source as possible.

A few other factors that affect the accuracy of the measurement using the pancake probe are

- the probe will likely be calibrated against ^{137}Cs photons (662 keV) and may not provide an energy-independent response so that some error in dose rate may occur, especially for the energies exceeding 2 MeV;

- the side walls of the probe may attenuate some photons entering from shallow angles and depress the response, and

- depending on the origin of the contaminated oil some of the activity could be associated with small particulates in the oil. If this might be the case you should try to ensure reasonably uniform distribution of the cobalt throughout the volume, possibly by stirring or other mixing procedure.

Expressions similar to but somewhat more complex than equation 1 may also be obtained for measurement points on the curved surface of the container, or for other possible geometries. You can find such solutions discussed and presented in various sources. One useful text is by Shultis and Faw titled *Radiation Shielding*, published in 1996 by Prentice Hall. Another older but helpful source, if you can find it available, that presents various regular geometry sources with acceptable approximate solutions is the *Engineering Compendium on Radiation Shielding*, Vol. 1, Ed. by Jaeger et al., Springer – Verlag, New York; 1968. The Microshield code is very easy to use, has an extensive library of radionuclides, and handles most common geometries. It costs close to $1,500 but may be worthwhile for a company or individual who performs a lot of these types of calculations or deals with other shielding problems involving regular geometries.

If you have an appropriately calibrated gamma spectrometry system available an alternative procedure would be to count a large volume sample (usually 1 to 4 liters) in a Marinelli-type beaker that is designed to fit around the detector.

Good luck in your measurements.

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